RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, AUGUST 2021

FIRST YEAR [BATCH 2020-23]

Date : 12/08/2021 **COMPUTER SCIENCE (HONOURS)**

Time : 11 am – 1 pm **Paper : IV [CC4]** Full Marks : 50

Answer **any five** of the following questions: [5×10]

1. a) If A, B, C are non empty sets, then prove that

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

b) Show that the mapping $f : R \rightarrow R$ defined by $f(x) = \text{Cos}(x)$, $\forall\ x \in R$ is neither one-one nor onto. Modify the domain and co-domain of this mapping so that it may be both one-one and onto.

c) Let S be the set of all positive divisors of 60. Define a relation '$\leq$' on S by " $x \leq y$ if and only if x is divisible by y " for $x, y \in S$. Prove that (S, $\leq$) is a poset. [3+4+3]

2. a) Let $G=\{(a, b) : a, b \text{ are rational, } b \neq 0 \}$. Prove that (G, o) is an abelian group where o is defined by (a, b) o (c, d) = (ad + bc, bd) for all (a, b), (c, d) in G.

b) Define Ring.

c) A kabaddi team play at least one match in a day. If the team play 45 matches in a month of 30 days, then show that they play exactly 14 matches in some consecutive days. [4+2+4]

3. a) How many words of 3 consonants and 2 vowels can be formed, if they are chosen from 6 consonants and 5 vowels?

b) Find a closed form for the generating function for the following sequence.

0, 1, -2, 4, -8, 16, -32, 64, . . .

c) Find the solution of the following recurrence relation: -

$$a_n = 6a_{n-1} - 9a_{n-2} + n3^n$$ [2.5+2.5+5]

4. a) In a town according to clinical test report, 1 in 1000 of the population is a carrier of the disease. Suppose also that the probability that a carrier tests negative is 1.5 %, while the probability that a non-carrier tests positive is 3 % (A test achieving these values would be regarded as very successful). If a patient has just had a positive test result , then what is the probability that he/she is a career ? Also find the respective probability, if the patient just had a negative test report.

b) Define Normal distribution.

c) Calculate Mean and Variance of X, if X represents the outcome when a fair dice is rolled. [4+2+4]

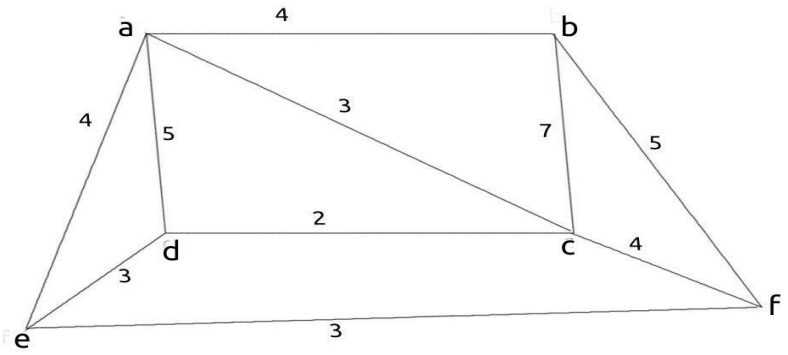
5. a) Prove that if a graph has odd order and is regular of degree $d \geq 1$, then it is not bipartite.

b) Prove that an Euler graph cannot have a cut-set with an odd number of edges.

c) Find the edge connectivity of the complete graph with n vertices.

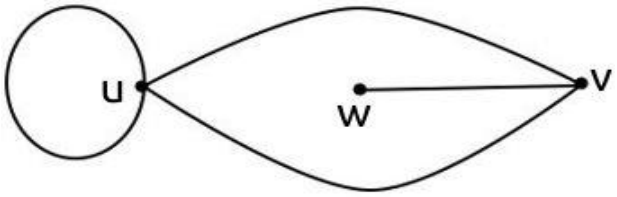
d) State and prove Euler theorem for planar graph. [2+2+2+4]

6. a) Using Kruskal's algorithm find out the shortest spanning tree for the following graph. Explain the necessary steps.



b) For a perfect binary tree with n vertices, find the number of pendant vertices.

c) Draw the geometric dual of the following graph, and also check whether the given graph and its dual are isomorphic to each other or not. [4+2+4]



7. a) Draw the Hasse-diagram for the poset (S, ⊇), where S is the set of all subsets of {1, 2, 3} and the relation '⊇' refers to its usual meaning.

b) In how many different ways can 18 identical balls be distributed among 4 persons?

c) Explain ring sum of two graphs with example.

d) Prove that every cyclic group is abelian group.

e) How many simple labelled graphs can be drawn with n vertices? [2.5+2+2+2.5+1]

———— × ————